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THESIS

DECOY EFFECTIVENESS IN A MULTIPLE SHIP
ENVIRONMENT

by

Cengiz Sengel

September 1988

Thesis Advisor: Edward B. Rockower

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In this problem, we allow the number of ships, number of decoys, and the values of ships to vary. We also vary the values of the hard kill probabilities, the splash probabilities of decoyed missiles, probabilities that a missile's lock is broken by seduction decoys and the quality factor of the distraction decoys.

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Decoy effectiveness in a multiple ship environment

by

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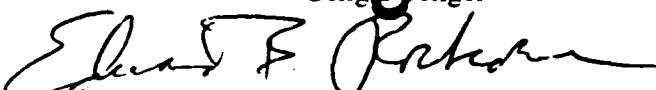
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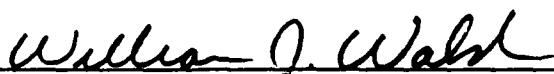
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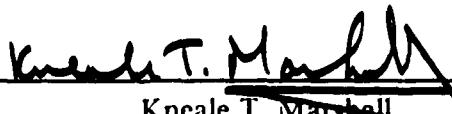
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ABSTRACT

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In this problem, we allow the number of ships, number of decoys, and the values of ships to vary. We also vary the values of the hard kill probabilities, the splash probabilities of decoyed missiles, probabilities that a missile's lock is broken by seduction decoys and the quality factor of the distraction decoys.

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Anti-ship Cruise Missile

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DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	

iii

TABLE OF CONTENTS

LIST OF FIGURES

I. INTRODUCTION

A. THE PROBLEM

The problem considered here is to defend a multiple-ship Battle Group (BG) against anti-ship cruise missile (ASCM) attacks. For this reason, a multiple-ship missile attack model has been developed to study the effect of Electronic Warfare (EW) expendable decoys, which may increase the survivability of a single ship or a battle group. The effectiveness of a single decoy, or of a group of identical or different decoys, employed during an ASCM attack on a BG may be degraded as a result of missile diversion onto neighboring own-force or allied ships in the BG.

The model that we present in Chapter III is intended to investigate how expendable decoys may increase survivability. The emphasis is on simplicity consistent with representing the multi-ship situation rather than on complete realism. Individual systems may have peculiarities that may make some of the assumptions in the model invalid.

In this study it is assumed that the BG is being attacked by one or more guided missiles. For each individual missile, an attack is assumed to consist of two phases:

1. Target acquisition by the missile seeker or guidance system.
2. Missile homing to impact.

For an individual missile to successfully engage a target, it must complete both phases. Expendable decoys can disrupt either of these phases. If the missile cannot be distracted during the target acquisition phase then it might be " seduced " in its homing process (breaking the acquisition of the missile seeker). The missile might then be diverted onto another ship, or an empty zone (occupied by no ships), or the diverted missile might " splash-down " to the ocean (missile might divert not only from its course but also it might divert to an altitude closer to the ocean level). Hence the missile might " splash " into the ocean as a result of losing its altitude after seduction. Also, the ship air defense guns might shoot down the missile. If the missile is not seduced then it might

1

be shot down by ship air defense guns, or if it is not shot down by ship guns it may hit a ship in the BG.

For many missiles vs many targets (ships), it is assumed that the missiles act independently and their errors are not correlated. It is also assumed that the effects of the decoys are independent. These assumptions of independence may hold in some operational circumstances but in others they may not hold.

The methods of defense against ASCM attacks for a BG that has at least one or two high-value ships and various combat and escort ships taken into account are:

1. Hard kill by ship guns or antimissile missiles.

2. Soft kill by using EW assets (splash down or diversion by using decoys).

In this study, we developed a simple model that enables us to compare different factors, such as effectiveness of different types of decoys (seduction and distraction), the effect of changing the numbers and types of decoys, decoys vs hard kill, and non-linear interactions caused by diversion onto friendly ships in a multiple-ship BG.

In this study, the following conditions hold for ASCM attacks on a multiple-ship BG environment situation:

1. Effectiveness is measured as survivability of the entire BG; not only for a single ship.

2. Multiple-ship environment; one or two-high value ships such as aircraft carriers, battle ships, or supply ships in or near the center of the BG. Low-value ships, such as destroyers, are far from the center of the BG.

3. Nonlinearities; ship-missile-ship interactions are important, not just one ship-missile/missiles interaction.

4. Decoys can be expendable decoys deployed from the ships for defending the ship itself, or the BG, against incoming ASCM attacks. Two types of expendable decoys are considered:

   a. Distraction decoys provide alternate targets for the missile during the target acquisition process.

   b. Seduction decoys disrupt the homing process.

Since this is an unclassified study, some additional assumptions regarding performance parameters are made to exercise the model. The model, computer code, and sensitivity studies can be readjusted for special conditions. Hence, the model and sensitivity studies give results that depend on unclassified information.

II. BACKGROUND

A. BATTLE GROUP (BG)

As depicted in Figure 1, an idealized BG is a group of ships that has at least one or two high-value ships and other low-value ships.
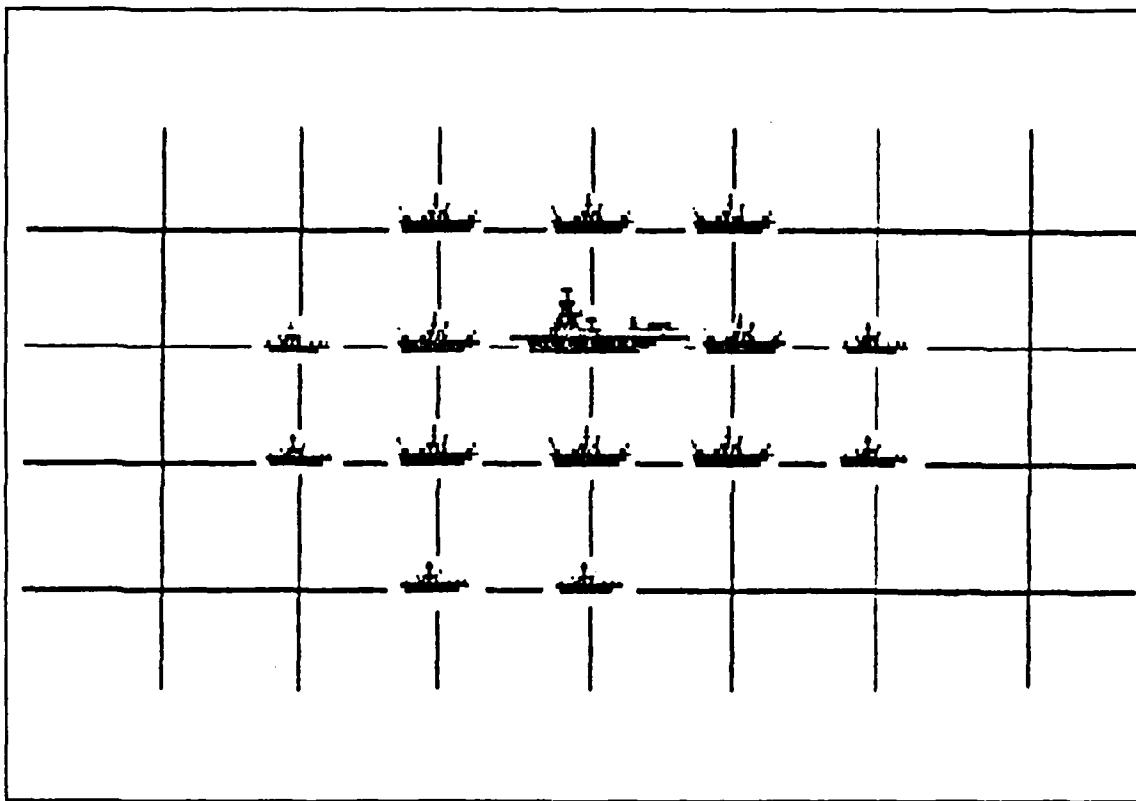

Figure 1.   An idealized battle group

The approximate number of total ships in the group is 8-15, which can be divided into two categories of ships based on their importance. These categories are:

1. High-value ships
2. Low-value ships

High value ships may be aircraft carriers (CV's), Battle ships (BB's), and supply ships (AOE's). Low value ships may be guided missile cruisers (CG's), destroyers (DD's), and frigates (FF's). The values of the ships in a BG may be related to the mission of the BG , location of the operation scene, and other factors that affect the BG scenario.

The basic weapons of a BG for defending itself against ASCM attacks are:

1. Guns.

2. Antimissile missiles.

3. Electronic Warfare assets, such as electronic counter measures

(ECM), electronic counter counter measures (ECCM), and decoys for seduction or distraction.

In this study we are primarily interested in the weapons for hard kills (guns, anti-missile missiles) and decoys for soft kills.

Mission types for a BG usually require it to remain on the ocean for a long period of time and to conduct some independent operations. Generally, the main threats to a BG come from the air, under the water, and the surface. But, in modern warfare, the most likely unexpected method of attacking a BG can be from under water or air. Today most conventional submarines may use ASCM's to attack surface targets. Hence we discuss ASCM attacks from different platforms.

B. ANTISHIP CRUISE MISSILES (ASCM'S)

ASCM attacks on a BG may come from any direction relative to the course of the BG. ASCM's can follow different flight patterns. Two possible patterns can be seen in Figure 2. The processes that an ASCM goes through in its flight pattern are:

1. Target acquisition process.

2. Homing process to impact.

The target acquisition process for an ASCM includes tracking, locating, identification and classification of the target; calculations for fire control; launching the missile; flight until the final phase, which includes precise tracking to the prelocated position of the target (homing to impact). The target acquisition process can vary for different types of missiles. We assume that this process occurs approximately from launching to 6000 - 20000 yards from the target that the anti-ship cruise missile (ASCM) is aimed at. The range of this process for a missile depends upon the characteristics of the guidance system of the missile.

4
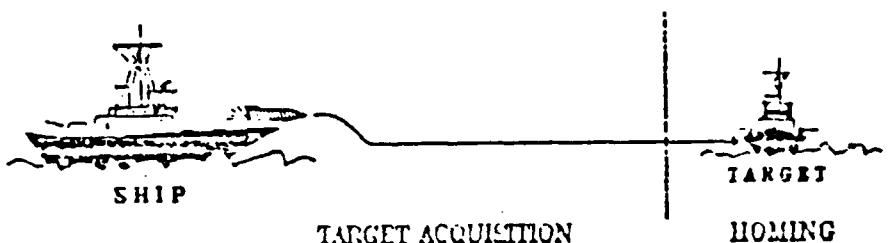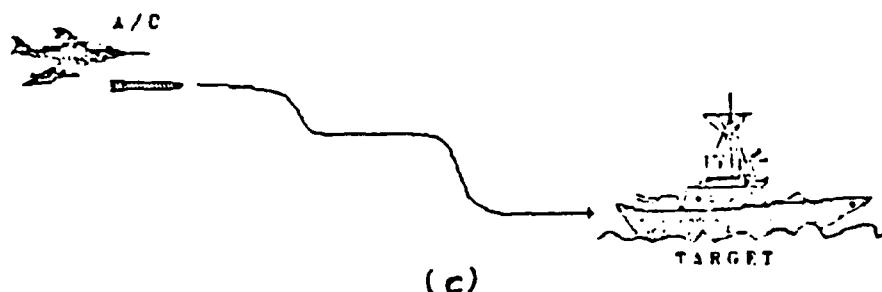
Figure 2.   Target acquisition and homing processes of different ASCM's

The homing process to impact for an ASCM starts at the end of the target acquisition process and continues to impact or miss of the target. The homing process occurs in relatively short distances compared to the target acquisition process. Although it varies from missile to missile, we assume that homing process can occur around 6000 - 20000 yards.

The speed of the incoming missile can vary from mach 1.5 to mach 2.5 for different types of missiles.

C. INTERACTIONS, ASSUMPTIONS, AND THE NATURE OF THE PROBLEM

The speed of the BG is usually from 10 to 30 knots. Considering the area that is covered by a BG, and comparing the speeds of the BG and the ASCM, we can assume that because the speed factor of the BG is small compared to the speed factor of the

5

incoming missiles, speed does not have a major effect on avoidance maneuvers of the BG against ASCM's. For this reason we can assume that the BG is almost fixed during the attack. Hence, we consider that the distance traveled by the ASCM in the target acquisition process is longer than the distance traveled by the ASCM in the homing process. Thus, this assumption about the fixed location of the BG is even more justified in the homing process than the target acquisition process.

As previously mentioned, a BG that has different kinds and values of ships can be attacked by ASCM's launched by surface , air or underwater platforms. In each case ASCM's are the faster, most unexpected, precise, and preferred manner of attack against a BG. Each different type of missile usually shows almost the same flight characteristics.

Hard kill devices of the BG for defending itself against ASCM's are high speed small caliber air defense guns, such as 20 millimeters, 35 millimeters and 40 millimeters --- for example, the PHALANX system. Short range surface to air missiles (SAM's) are also used --- for example the SEA SPARROW air defense missile system. Since this study contains unclassified information, a hypothetical hard kill probability (PHK) of the BG for an incoming missile is chosen.

Soft kill devices (decoys) are of two types: distraction decoys and seduction decoys. The quality factor (Q) for distraction decoys can be defined as a measurement of the distraction ability of a distraction decoy. If a distraction decoy has $Q = 1.0$, it has the same effect as a real ship on a missile e.g. essentially the same Radar Cross Section (RCS). Quality factors take on values in the interval $[0,1]$

The probability of breaking the lock of a missile by a seduction decoy, (B), can be defined as a measurement of the seduction ability of a seduction decoy. After seducing an incoming missile, if the seduction decoy successfully breaks the lock of the incoming missile, the missile may divert or splash into the ocean. For different flight patterns there are different splash probabilities. For example, the flight pattern in Fig. 2.a has a higher splash probability than the one in Fig. 2.b

6

III. SURVIVABILITY OF BATTLE GROUP

A. THE MODEL TO CALCULATE THE PROBABILITY MASS FUNCTION VALUES (PMF) FOR NUMBER OF ENCOUNTERS.

An idealized BG, has approximately 8 - 15 ships. If we define N_0, A, and, R by

1. N_0 : number of ships in the BG,
2. A : area covered by the BG,
3. R : radius of the BG,

then the ship density (ρ) for the BG which covers area A is

$$\rho = \frac{N_0}{A} .$$

It seems reasonable to suppose the ships will maintain their distance from each other when an attack is imminent. Hence for simplicity of analysis we assume the ships are located on the intersections of the grid lines within the area A. Later we will allow the ships to be uniformly located on approximately *parallel lines normal to the missile tra-jectory*. The area per ship in the BG is clearly ($\frac{A}{N_0}$) which can be visualized as a square of side, ($\sqrt{\frac{A}{N_0}}$) , see Figure 3.a. Hence the approximate distance (D) between ships is

$$D = \sqrt{\frac{A}{N_0}} .$$

As seen in Figure 3.b, the random variable L is the distance which the missile travels as it traverses the BG. The number (N) of possible ship encounters by the missile can be approximated as the integer value closest to N where,

$$N = \frac{L}{\sqrt{\frac{A}{N_0}}} .$$
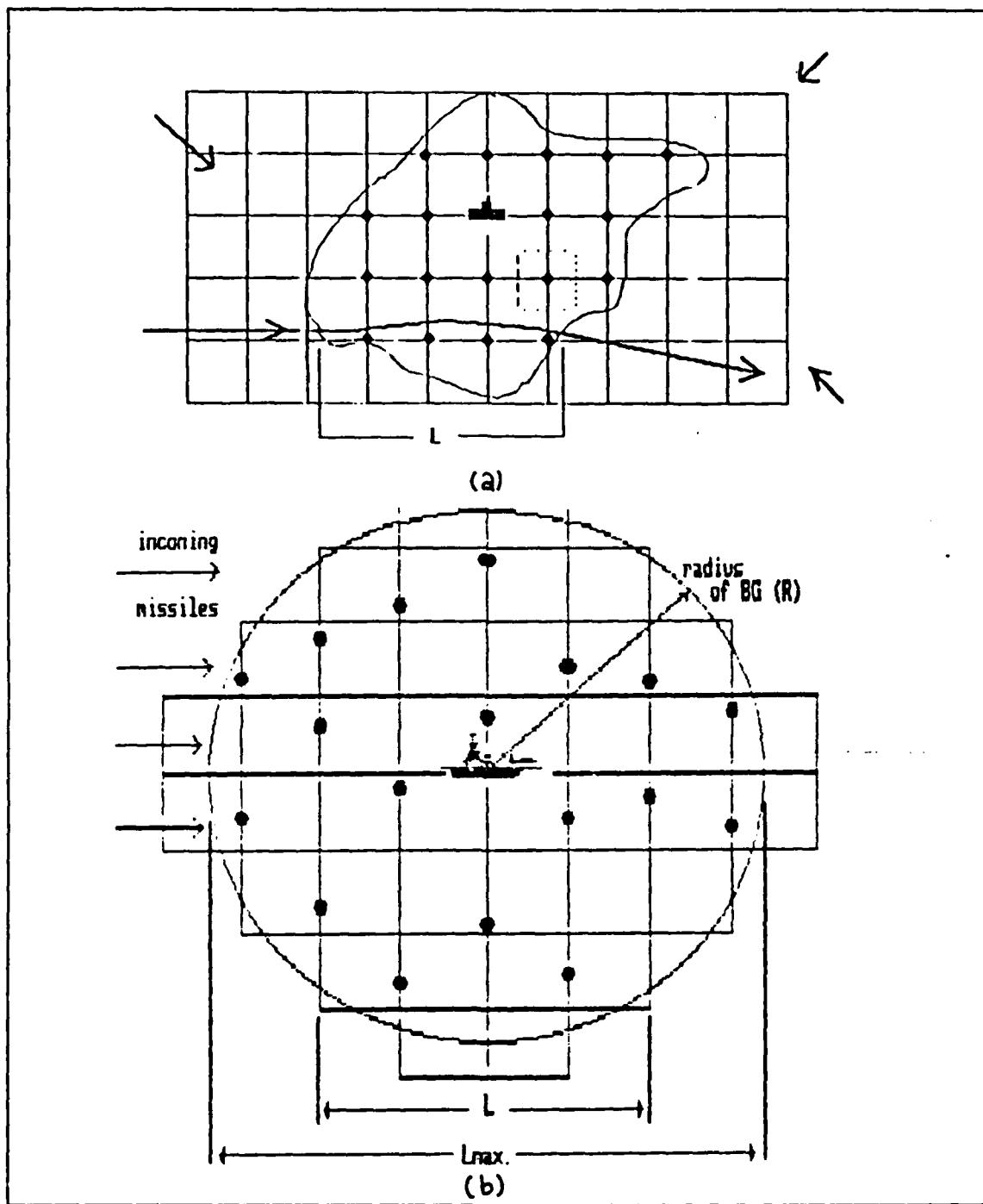
Figure 3. Missile attacks and locations of the ships in BG for the model

Lower case n, l, and, n′ will denote particular realizations of the r.v.'s N, L, and, N′ Since 2R is the maximum distance which a missile can travel within the BG, the maximum number of encounters will occur for L = 2R; this is given by

$$N_{max} = 2R\sqrt{\frac{N_0}{A}} \ .$$

We assume the missiles meet at least one line of ships in the BG. In other words we are not concerned about missiles which miss the BG. Hence the range of N is,

$$1 \leq N \leq 2R\sqrt{\frac{N_0}{A}} \ .$$

In the absence of intelligence reports and any detection of enemy missiles, a missile attack may come from any direction. Since there is a circular symmetry because of random direction of attacks, we assume the BG has a circular shaped formation. A circular formation can give more layer defense to the high-value ships in the BG by locating them in or near the center in case of missile attacks from any direction, and it can give efficient ultra high frequency (UHF) / visible communication conditions among all ships in the BG and the BG commander.

In a circular formation, the area (A) covered by the BG is

$$A = \pi R^2 \ .$$

So the maximum number of possible encounters is

$$N_{max} = 2R\sqrt{\frac{N_0}{\pi R^2}} \ = \ 2\sqrt{\frac{N_0}{\pi}} \ , \tag{3.1}$$

and for any distance L

$$N = L\sqrt{\frac{N_0}{\pi R^2}} \ . \tag{3.2}$$

Let the closest point of approach (CPA) to the center of the BG for an ASCM be the random variable, Y, which is uniformly distributed in | 0,R | From Figure 4 we see that Y, determines L, and therefore also N.

Let N′ be a random variable which is N rounded to the nearest integer. Then we can find its probability mass function values (PMF) based on the underlying uniform distribution of Y, for any given N_0 (the number of ships in the BG); see Figure 4.
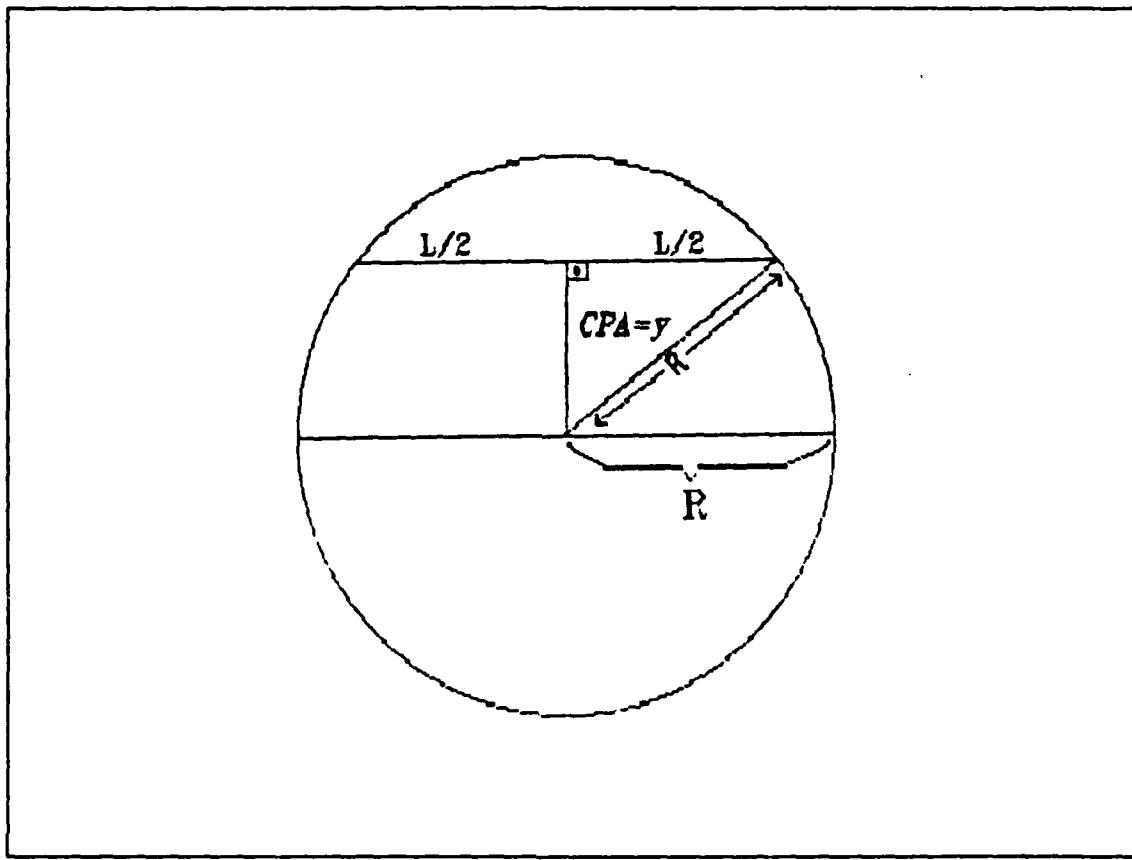
Figure 4.   Drawing for calculation of the eqn. 3.2 and 3.3

From Figure 4 we see that

$$L = 2\sqrt{R^2 - Y^2} \; ,$$ (3.3)

hence, using eqns. (3.1), (3.2), and, (3.3)

$$N = 2\sqrt{R^2 - Y^2} \sqrt{\frac{N_0}{\pi R^2}} \;\; = \;\; \frac{2}{R} \sqrt{N_0 \frac{R^2 - Y^2}{\pi}} \; .$$

If we take the square of the each side,

$$N^2 = \frac{4}{R^2} \frac{(N_0 R^2 - N_0 Y^2)}{\pi} \;\; = \;\; \left(\frac{4N_0}{\pi}\right) - \left(\frac{4N_0 Y^2}{\pi R^2}\right)$$

Solving for Y we find,

10

$$Y^2 = \left(\frac{4N_0}{\pi} - N^2\right)\left(\frac{\pi N^2}{4N_0}\right),$$

$$Y = R\sqrt{\left(1 - \left(\frac{N^2\pi}{4N_0}\right)\right)}.$$

Replacing the r.v.'s Y and N with y and n we obtain the corresponding relations between the realizations of the r.v.'s. The latter equations are used to obtain the derivative in the transformation formula

$$f_N(n) = f_Y(y)\left|\frac{dy}{dn}\right|,$$

hence, using $f_Y(y) = \frac{1}{R}$,

$$= \left(\frac{1}{R}\right)\left|\frac{-Rn\pi}{4N_0\sqrt{1 - \frac{n^2\pi}{4N_0}}}\right|,$$

and finally

$$f_N(n) = \frac{n\pi}{4N_0\sqrt{1 - \frac{n^2\pi}{4N_0}}}. \tag{3.4}$$

Part of the computer program which we discuss in Chapter 4 computes the probability mass values (PMF) for a discrete number of encounters using the PDF of n which was calculated in eqn. (3.4). As seen in Figure 5, the dotted line shows the points which are calculated by using eqn. (3.4) for $N_0 = 14$. Since there are a discrete number of layers in reality we find the probability mass values (PMF) for each discrete layer by calculating the sub portions of area under the dotted line which are; 0.5 to 1.5 for 1st. layer, 1.5 to 2.5 for 2nd. layer, 2.5 to 3.5 for 3rd. layer. For Figure 5 the maximum number of layers = 4.2 which is calculated using the eqn. (3.1) for $N_0 = 14$ in the BG. Since the maximum N' is the integer number closest to n, which is 4, we then use 3.5 to 4.2 for the probability of the 4th. layer.

Now let us name the small portions of the area under $f_N(n)$ as, $A_1, A_2, A_3, ..., A_i$, where i is the closest integer value to n_{max}. If we want to calculate the area under the dotted line

11

(PDF of n), which will be called P in this calculation, we can first calculate the A_0 which is the area under the $f_N(n)$ from 0 to 0.5,

$$A_0 = \int_0^{0.5} f_N(n) dn \; = \; \int_0^{0.5} \frac{n\pi}{4N_0 \sqrt{1 - \dfrac{n\pi}{4N_0}}} \, dn \; .$$

Then we can find the area of probability for PDF of n between 0.5 and n_{max} by calculating, (since $N \geq 1$, we eliminate probability associated with N' = 0.)

$$P = 1 - A_0$$

or we can find total area by calculating

$$P = A_1 + A_2 + A_3 + \dots + A_l \; ,$$

where

$$A_1 = \int_{0.5}^{1.5} \frac{n\pi}{4N_0 \sqrt{1 - \dfrac{n\pi}{4N_0}}} \, dn \; ,$$

$$A_2 = \int_{1.5}^{2.5} \frac{n\pi}{4N_0 \sqrt{1 - \dfrac{n\pi}{4N_0}}} \, dn \; ,$$

$$A_3 = \int_{2.5}^{3.5} \frac{n\pi}{4N_0 \sqrt{1 - \dfrac{n\pi}{4N_0}}} \, dn \; ,$$

$$\begin{matrix} '' & & '' \\ '' & & '' \\ '' & & '' \end{matrix}$$

$$A_l = \int_{l-.5}^{n_{max}} \frac{n\pi}{4N_0 \sqrt{1 - \dfrac{n\pi}{4N_0}}} \, dn \; .$$

12

NUMBER OF SHIPS IN THE BG (N) = 14

(Y-axis: PROBABILITY DENS./MASS VALUES OF NUMBER OF LAYERS; X-axis: NUMBER OF LAYERS OF SHIPS IN THE BG)
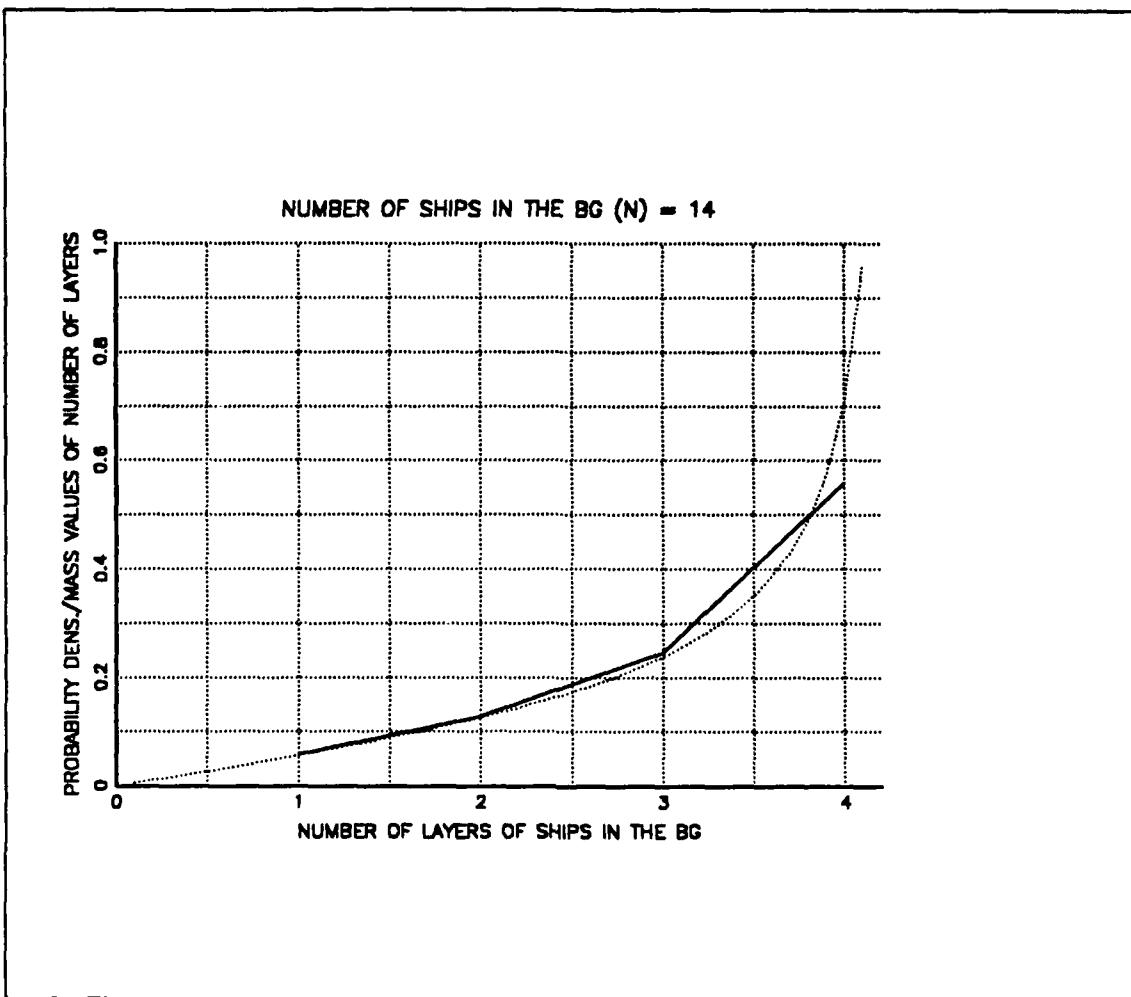
Figure 5. Calculation of the PMF values for # encounters per missile

Either of these two approaches should give the correct result for total probability under the PDF of $n \geq 0.5$. We shall use these two approaches to check the results of the computer program presented in Chapter 4.

After finding the total area under the PDF of $n \geq 0.5$, we normalize probability mass values of each layer with;

$$PMF_1 = \frac{A_1}{P} \quad ,$$

$$PMF_2 = \frac{A_2}{P} \quad ,$$

13

$$PMF_3 = \frac{A_3}{P} \quad ,$$

$$\begin{array}{cc} '' & '' \\ '' & '' \\ '' & '' \end{array}$$

and for ith. interval,

$$PMF_i = \frac{A_i}{P} \quad .$$

Now the sum of all the probabilities should be

$$PMF_1 + PMF_2 + PMF_3 + ... + PMF_i = 1 \quad ,$$

so that the number of possible encounters is an integer ≥ 1 rather than ≥ 0.

Figure 5 gives an example calculation of the normalized PMF values for the number of possible encounters when $N_0 = 14$. The dotted line shows the values of $f_N(n)$ and the solid line connects the normalized PMF value points for each discrete layer.

B. THE MODEL TO CALCULATE THE PROBABILITIES OF SHIP HIT, MISSILE KILL, MISSILE DIVERSION, AND SHIP SURVIVABILITY

1. Probability calculation for one layer

The probabilities of ship hit , missile kill, missile diversion, and ship survivability can be calculated by constructing a sample probability tree model for one layer. As seen in the tree diagram in Figure 6, when a missile arrives to a line of possible encounters it may meet one or more ships or no ships. We now assume that the ships are located randomly along the line perpendicular to the missile trajectory.
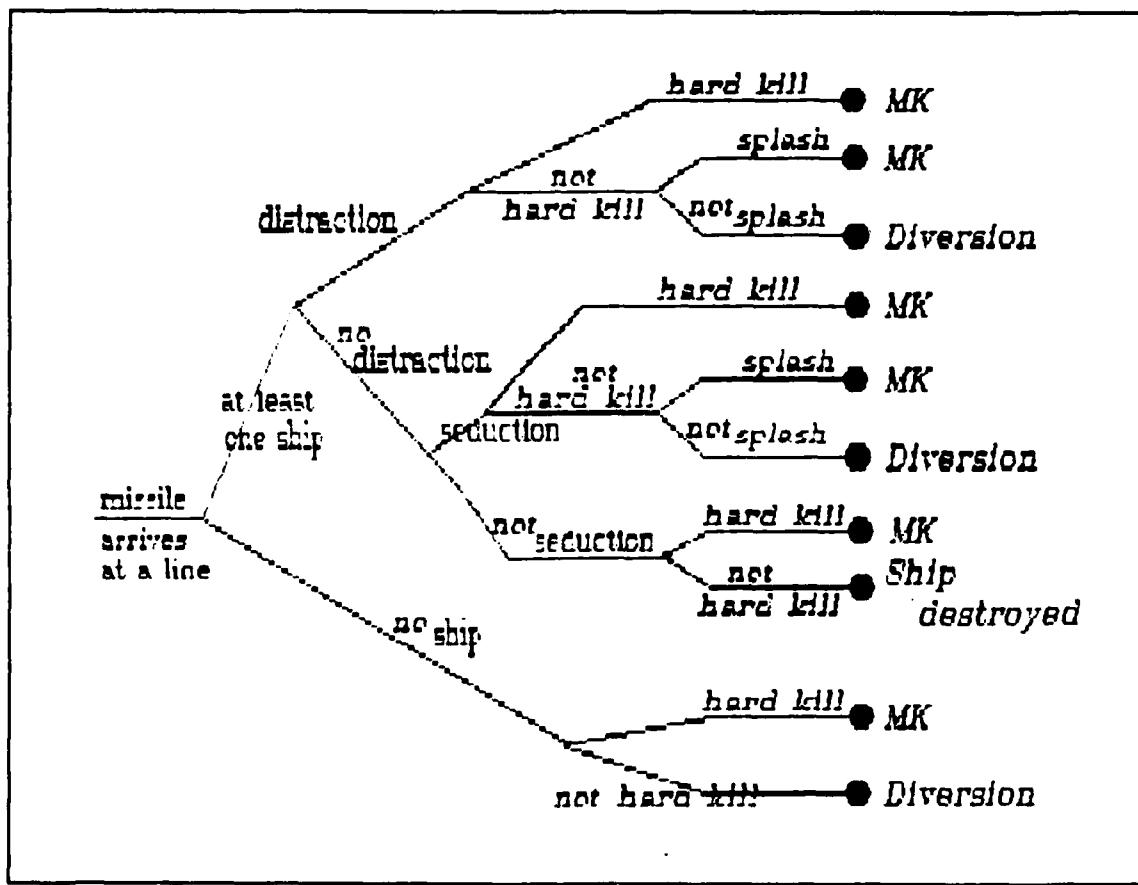
Figure 6.   Probability tree diagram for one layer of encounters

As can be seen in Figure 7, if α is the angular search beam of the missile, then the probability a missile does not detect a ship on the layer (PONS) is

$$PONS = e^{-2\alpha} ,$$ (3.5)

$\sqrt{A/N}$

2α

$2\alpha\sqrt{A/N}$

$\rho_{line} = \sqrt{N/A}$

• P_r [0 ships in missile beam-width] $\approx$ exp{-2α}
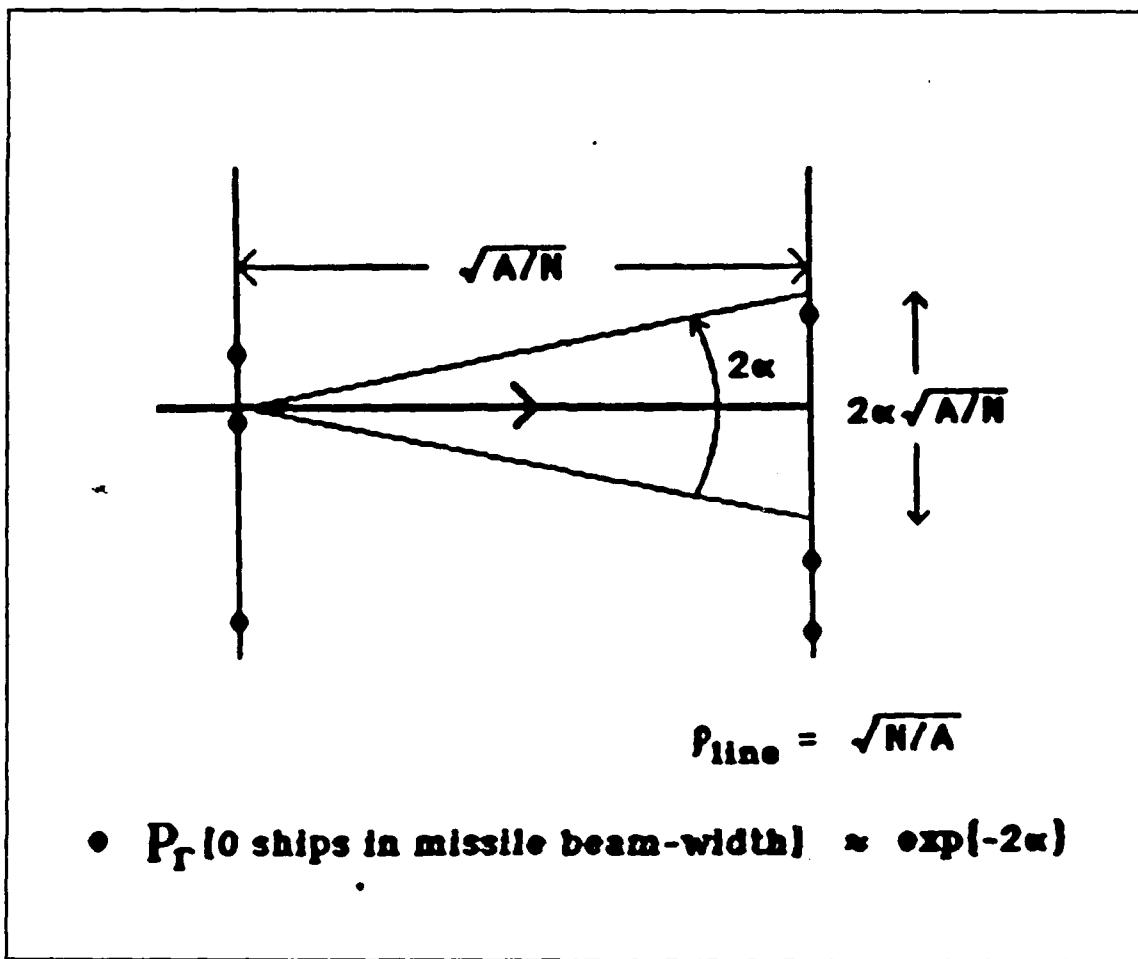
Figure 7.   Calculation for probability if missile meets at least a ship

which is based on the Poisson Approximation to the Binomial Distribution of the number of ships falling within the angular search beam width of the missile. On the other hand the probability a missile detects at least one ship per layer (POS) is obviously

$$POS = 1 - e^{-2\alpha} . \tag{3.6}$$

a.   Missile meets at least one ship

If a missile meets at least one ship, then it can be successfully distracted or not distracted by the ship using a distraction decoy with quality factor Q. The dis-

traction probability (POD), which is the ratio of the equivalent number of false targets (QD) to the total number of targets ($1 + QD$)[1] , and is assumed to be given by:

$$POD = (\frac{QD}{(1 + QD)}) , \qquad (3.7)$$

where D is the number of distraction decoys deployed by a given ship during the missile attack. On the other hand, the probability of no distraction of a missile given it meets a ship (POND) is

$$POND = (\frac{1}{(1 + QD)}) . \qquad (3.8)$$

Then:

If distraction occurs, then the missile can be destroyed by a hard kill; The probability of hard kill given the missile is distracted is

$$PMK_1 = (1 - e^{-2\alpha})(\frac{QD}{(1 + QD)})PHK . \qquad (3.9)$$

If the missile is not hard killed it can splash down and be destroyed (soft kill) or not splash down and instead be diverted after the distraction of the missile occurs. The probability of splash down of a missile = PSPL. If splash occurs obviously we can accept this as a missile soft kill. On the other hand, even if it is not splashed it is still diverted. In this case the probability of missile kill given the missile is distracted is

$$PMK_2 = (1 - e^{-2\alpha})(\frac{QD}{(1 + QD)})(1 - PHK)PSPL , \qquad (3.10)$$

and the probability of diversion (PDIV) given the missile is distracted is

$$PDIV_1 = (1 - e^{-2\alpha})(\frac{QD}{(1 + QD)})(1 - PHK)(1 - PSPL) . \qquad (3.11)$$

If a missile is not distracted with the probability calculated in eqn. (3.7) it can be seduced or not seduced by the BG using seduction decoys. In this case the probability of seduction (PSED) is

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1 Center for Naval Analysis (CNA) 86-1737 / 15 September 1986

$$PSED = (1 - (1 - B)^{MS}) \ , \tag{3.12}$$

and, the probability of no seduction is

$$PONS = (1 - B)^{MS} \tag{3.13}$$

respectively, where

  1. B $=$ Probability of breaking the lock of a missile for a seduction decoy in the BG.
  2. MS $=$ Number of seduction decoys in per ship.

If seduction occurs the missile can be destroyed by hard kill or not destroyed by hard kill. The probability of missile is hard killed given the seduction occurs is

$$PMK_3 = (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})(1 - (1 - B)^{MS})PHK \ . \tag{3.14}$$

The probability of no hard kill given that seduction occurs is

$$PMK_4 = (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})(1 - (1 - B)^{MS})(1 - PHK)PSPL \ , \tag{3.15}$$

and the probabability of diversion given the seduction occurs is

$$PDIV_2 = (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})(1 - (1 - B)^{MS})(1 - PSPL) \ . \tag{3.16}$$

If seduction does not occur, then the probability of hard kill given the seduction does not occur is

$$PMK_5 = (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})((1 - B)^{MS})PHK \ , \tag{3.17}$$

or if it cannot be hard killed it may hit a ship so the probability of a missile hitting a ship given the seduction does not occur is

$$PSHI = (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})((1 - B)^{MS})(1 - PHK) \ . \tag{3.18}$$

18

b. Missile meets no ship

If a missile meets no ships, then it can still be destroyed by hard kill, then the probability of hard kill given the missile meets no ships is

$$PMK_6 = e^{-2\alpha}PHK ,$$ (3.19)

and the probability of diversion given the missile meets no ships is

$$PDIV_3 = e^{-2\alpha}(1 - PHK) .$$ (3.20)

From all of these equations we calculated above, we can calculate the total missile kill probability (TPMK) per one line of encounters by using the equations.

$$TPMK = PMK_1 + PMK_2 + PMK_3 + PMK_4 + PMK_5 + PMK_6 .$$ (3.21)

If we write the whole equation for TPMK;

$$TPMK = (1 - e^{-2\alpha})(\frac{QD}{(1 + QD)})PHK + (1 - e^{-2\alpha})(\frac{QD}{(1 + QD)})(1 - PHK)PSPL$$

$$+ (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})(1 - (1 - B)^{MS})PHK + (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})$$

$$(1 - (1 - B)^{MS})(1 - PHK)PSPL + (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})((1 - B)^{MS})PHK$$

$$+ (e^{-2\alpha})PHK .$$ (3.22)

The probability of ship hit in the BG (PSHI) is again,

$$PSHI = (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})((1 - B)^{MS})(1 - PHK) ,$$ (3.23)

so survivability of BG (the probability that all ships survive the attack) as a measure of effectiveness for one line (PSSUR) is

$$PSSUR = (1 - (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})(1 - B)^{MS}(1 - PHK)) .$$ (3.24)

The total diversion probability (TPDIV) is then

$$TPDIV = PDIV_1 + PDIV_2 + PDIV_3$$

or

$$TPDIV = (1 - e^{-2\alpha})(\frac{QD}{(1 + QD)})(1 - PHK)(1 - PSPL) + (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})$$

$$(1 - (1 - B)^{MS})(1 - PSPL) + (e^{-2\alpha})PHK \ . \tag{3.25}$$

Equations 3.21, 3.22, 3.23, 3.24 are calculated on the probability tree diagram for one line of encounters in Appendix B. The reader can refer to Appendix B to get an overall idea for understanding better.

2. Probability calculation for all the layers

We have shown how to calculate the maximum number of lines using eqn. (3.2), the probabilities of any ship being hit in the BG, missile kill, missile divert, and survivability, for one line of encounters. Now we can find the probabilities conditioned on the number of layers (or potential encounters with ships) which make up the whole BG. For these calculations we shall use a probability tree, which can be seen in Figure 8, for all the layers in the BG, and we will be using the equations that we calculated for one layer encounters.

As we can see in Figure 7, for each line the missile may hit a ship or get diverted or get " killed ". If a missile hits a ship or gets killed, the threat of the missile is terminated. But if a diversion occurs the same conditions and probability calculations take place again and again until it passes the last line of encounters within the BG. For the last line, diversion is equivalent to the termination of the missile.

By using the equations in Part 1 and also using Figure 7 and Figure 8 we can find the following probabilities for a BG:

Probability of missile destruction for a BG (PMKBG) averaged over the number of lines of encounters for a given number (N) of ships is

$$PMKBG = \sum_{i=1}^{n} Pr\,[MK \mid i\ layers]\,PMF_i \ . \tag{3.26}$$

For instance by adding the probabilities for mutually exclusive (M.E.) events we have:

20
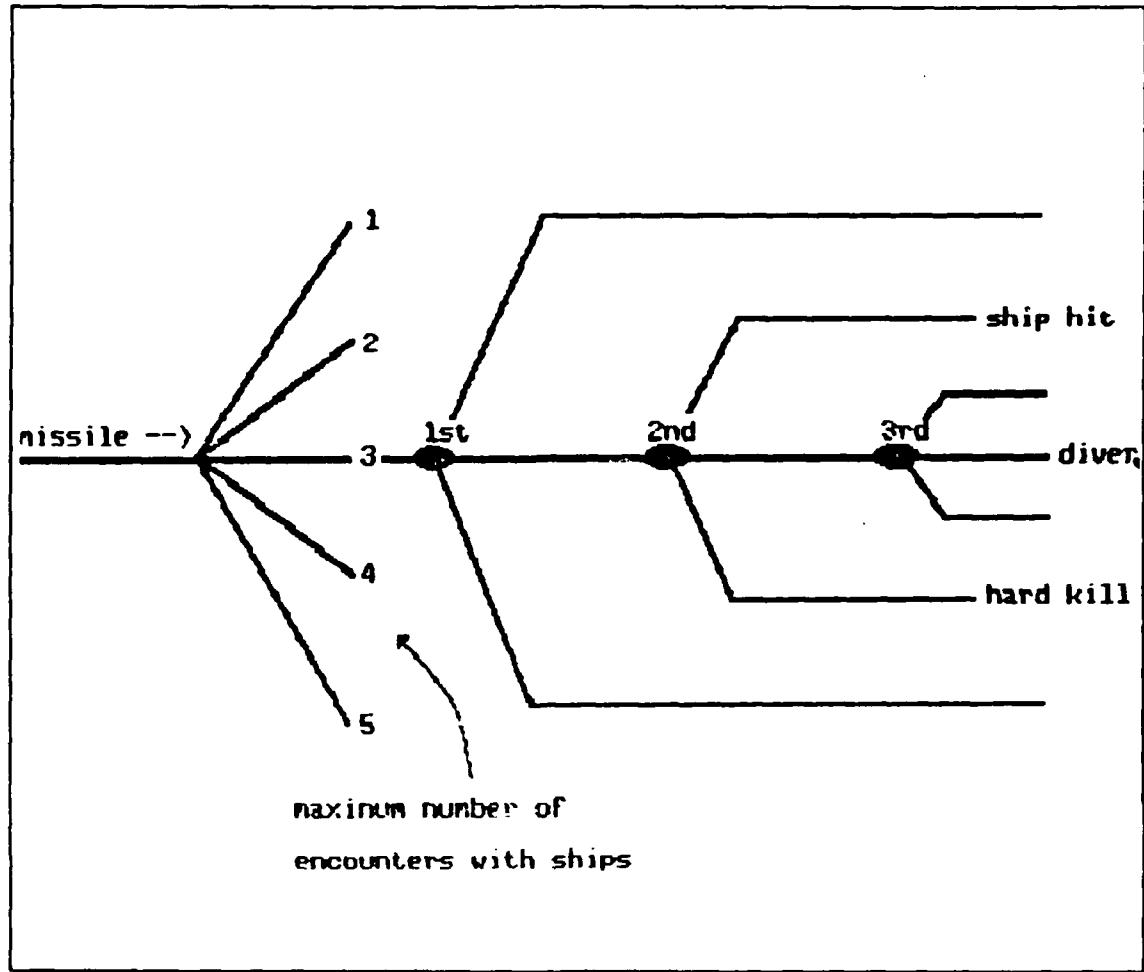
Figure 8.   Probability tree diagram for all the layers in the BG

$$Pr[MK \mid 3 \ layers] = PMK + (PDIV) PMK + (PDIV^2) PMK \ .$$

Probability of a ship getting hit for a BG (PSHBG) which has n lines of encounters for a given number (N) of ships is

$$PSHBG = \sum_{i=1}^{n} Pr \ [SHI \mid i \ layers] \ PMF_i \ . \tag{3.27}$$

For instance by adding probabilities for M.E. events we have,

$$Pr[SHI \mid 3 \ layers] = PSHI + (PDIV) PSHI + (PDIV^2) PSHI \ .$$

21

Probability of diversion of a missile for a BG (PDIVBG) which has n lines of encounters for given number (N) of ships is

$$PDIVBG = \sum_{i=1}^{n}(PDIV)^i PMF_i .$$ (3.28)

Also we can calculate the probability of ship survivability (PSSBG) as measure of effectiveness (MOE) for the BG.

$$PSSBG = (1 - PSHBG) .$$ (3.29)

We will be using the probabilities we calculate in eqns. 3.26, 3.27, 3.28 and 3.29 for drawing the graphs to understand the different relationships between the survivability of the BG and factors such as the number of each different type of decoy, the number of ships in the BG, the number of missiles attacking the BG, the distraction decoy Q factor, the seduction decoy B, factor, etc.

To illustrate these relationships we can calculate probabilities in eqns .3.26, 3.27, 3.28 and 3.29 by using the FORTRAN 77 program in the Appendix. After we run the program by changing the different parameters we get the example results as Figures 9 - 17.

As seen in Figure 9 we run the program with the parameters:

1. Number of ships in the BG (N_0) = 5...50.
2. Number of distraction decoys per ship (DI) = 1.
3. Number of seduction decoys per ship (MS) = 1.
4. Hard kill probability of BG (PHK) = 0.25.
5. B factor of seduction decoys = 0.2.
6. Q factor of distraction decoys = 0.2.
7. Splash probability of missile (PSPL) = 0.05.
8. Alpha angle of missile (α) = 15°.

and later we get the ship survival probabilities as the number of ships in the BG are increasing. We can easily see from the graph that when the number of ships increases in the BG, survivability of ships decreases. Also ship survivability of BG has a more negative slope (decreasing survivability) for N_0 = 5 to 15 than N_0 = 15 to 50 (50 can be thought of as very large = infinite number of ships for BG). When the number of ships goes to infinity, then the probability of survivability of BG is almost constant.

22
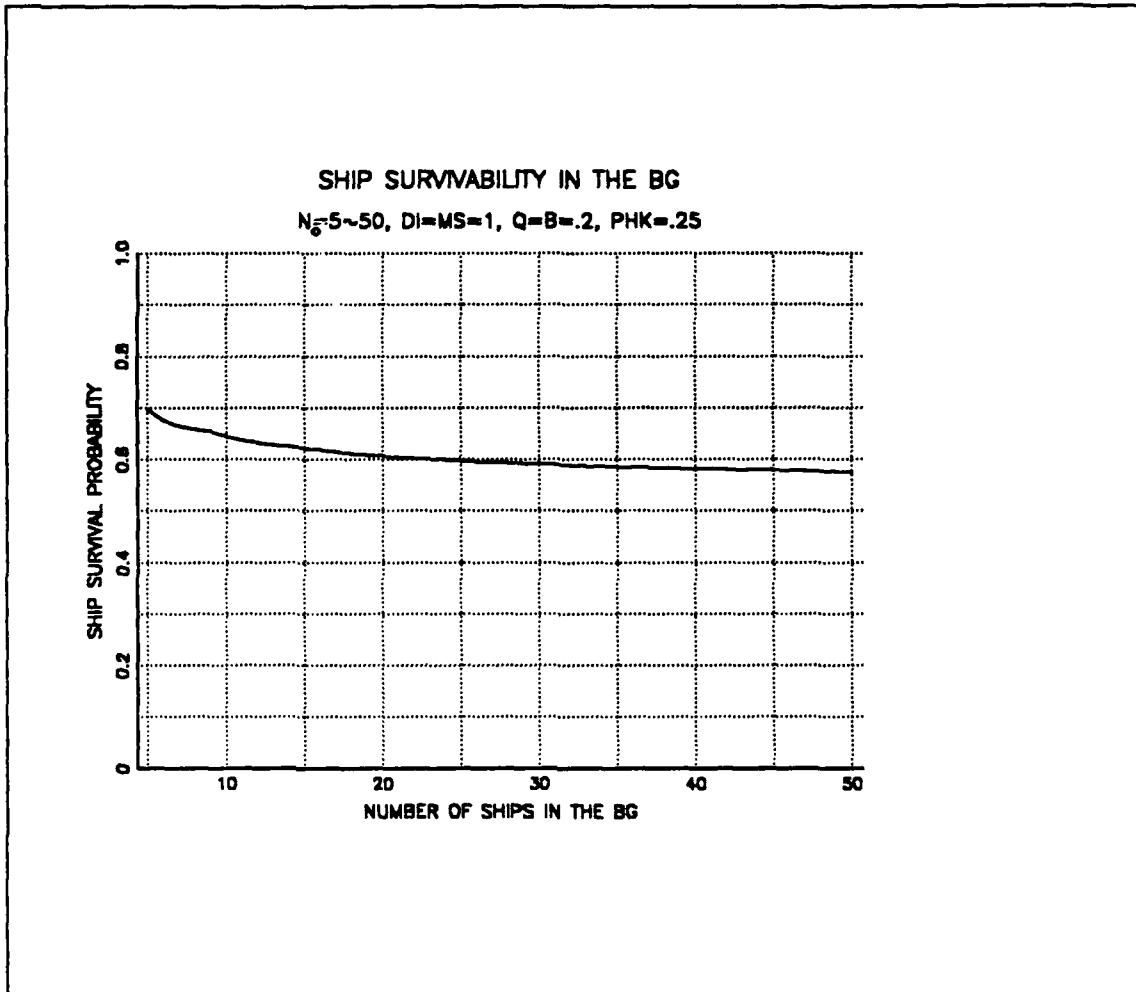
Figure 9. Probability of no hits, while number of ships varies

If we look at Figure 10 and the Figure 11 we can easily see that the different types of decoys affect the survival probability differently. In Figure 10:

1. The number of ships (N_0) varies from 5-30.

2. Number of distraction decoys (DI) varies from 1-5.

We get five different survival curves for BG. while the other parameters remained unchanged and are,

3. Number of seduction decoys per ship (MS) = 1.

4. Probability of hard kill level of BG (PHK) = 0.25

5. Splash probability of the missile (PSPL) = 0.05

6. B factor of seduction decoys = 0.2

23
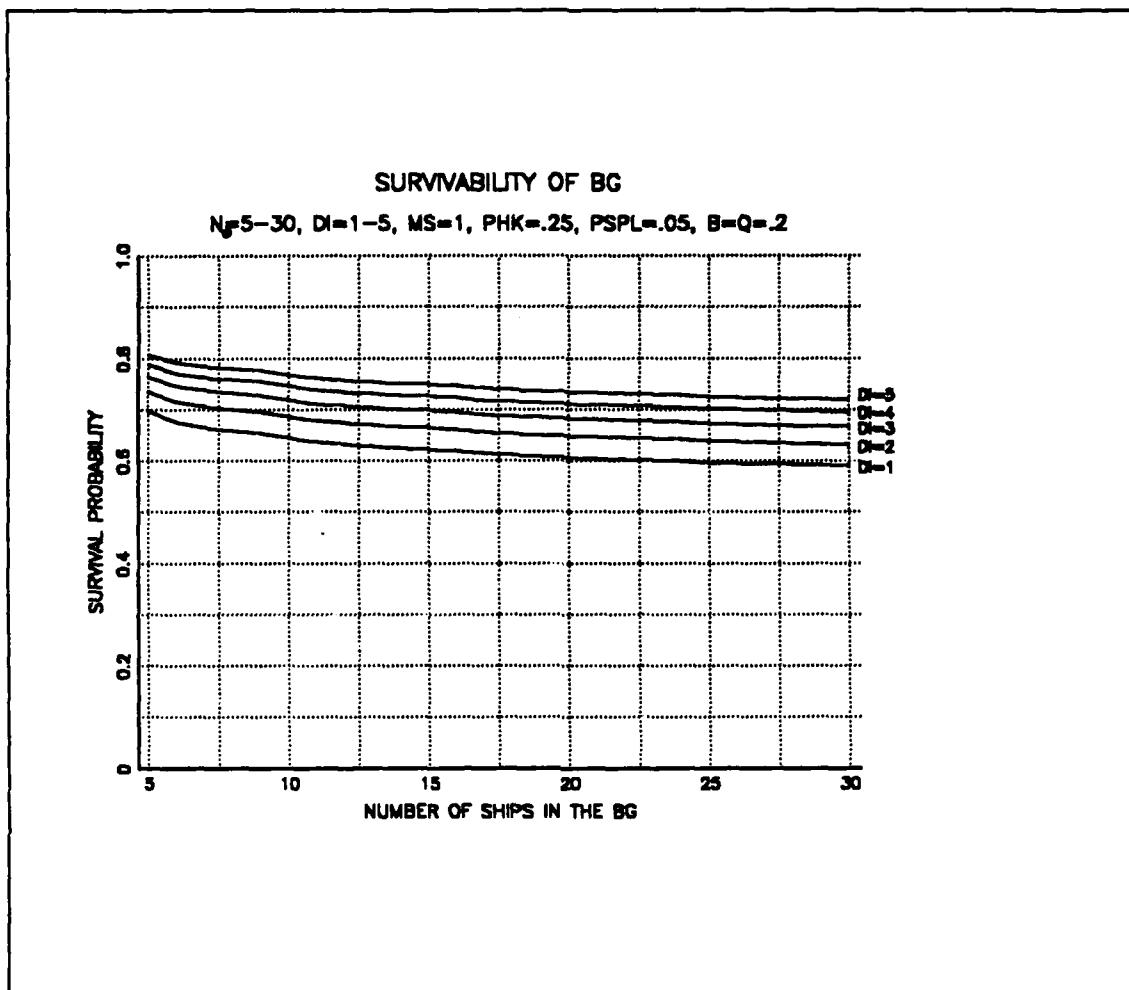
Figure 10.   Probability of no hits, distraction decoys vary from 1 to 5

       7. Q factor of distraction decoys = 0.2

       8. Alpha angle of the missile (α) = 15°

In Figure 11:

       1. Number of distraction decoys (DI) = 1.

       2. The number of seduction decoys (MS) varies from 1-5.

We get five different survival curves for the BG. while the other parameters remained unchanged and are the same as above. As we see in Figures 10 and 11, different types of decoys affect the survivability. In this case the number of seduction decoys has slightly more effect than the distraction decoys on survivability of a BG.

SURVIVABILITY OF BG

$N_0=5-30$, MS=1-5, DI=1, PHK=.25, PSPL=.05, B=Q=.2
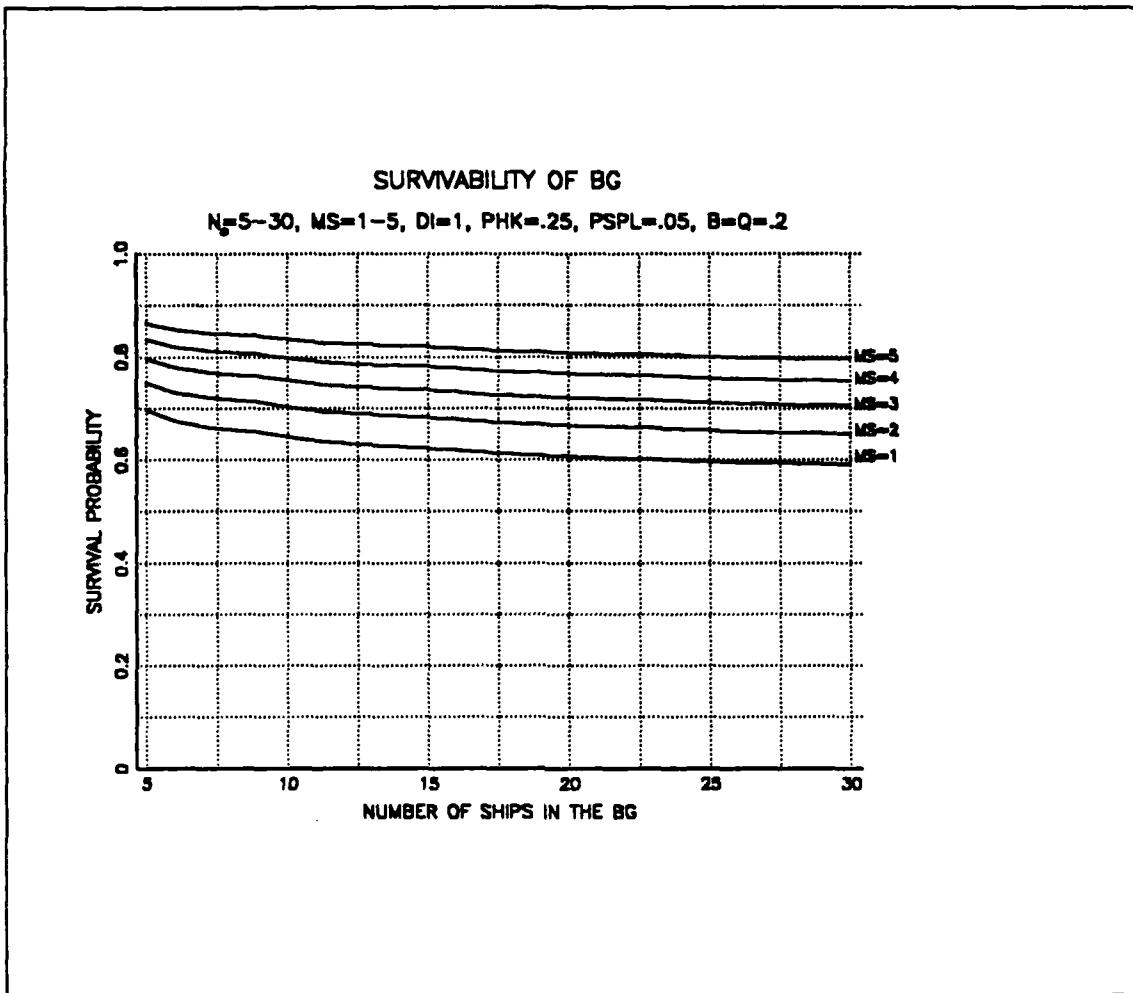
Figure 11. Probability of no hits, seduction decoys vary from 1 to 5

If we change the number of decoys, but we do not change the number of ships in the BG, we can get Figure 12 and Figure 13. In Figure 12:

1. Number of distraction decoys (DI) varies from 1-5.

2. The number of ships (N_0) = 5.

3. Number of seduction decoys per ship (MS) = 1.

4. Probability of hard kill level of BG (PHK) = 0.25.

5. Splash probability of the missile (PSPL) = 0.05.

6. B factor of seduction decoys = 0.2.

7. Q factor of distraction decoys = 0.2.

8. Alpha angle of the missile (α) = 15°.

SURVIVABILITY OF BG

N=15, DI=1-5, MS=1, PHK=.25, PSPL=.05, B=Q=.2
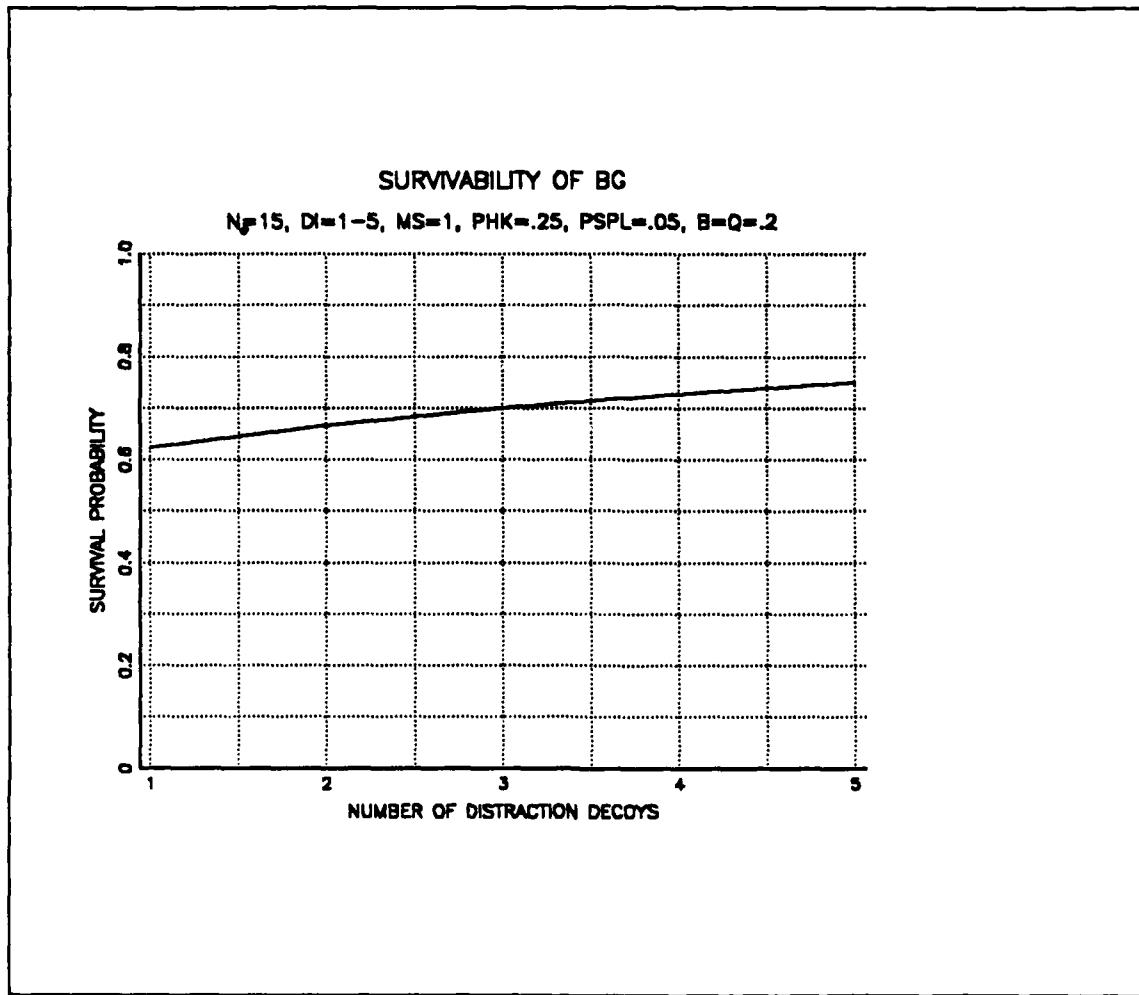
Figure 12.   Survivability of BG while num.of distraction decoys varies

In Figure 13, we modify the first and third parameters so that:

        1. Number of distraction decoys (DI) = 1.

        3. Number of seduction decoys (MS) varies from 1 to 5.

In these two figures, when the seduction decoys were changed, we obtain a   slightly higher survival probability compared to when the number of distraction decoys were changed. The benefit of expendables are apparent from these figures.

        The number of missiles attacking the BG is also an important factor for survivability, which is:

$$Pr \left(\ Survival\ given\ M\ missile\ attacks \ \right) \ = \ Pr \left(\ Survivability\ of\ BG \ \right)^{M} \ , \quad M = 1,2,...$$

SURVIVABILITY OF BG

N=15, MS=1-5, DI=1, PHK=.25,PSPL=.05,B=Q=.2
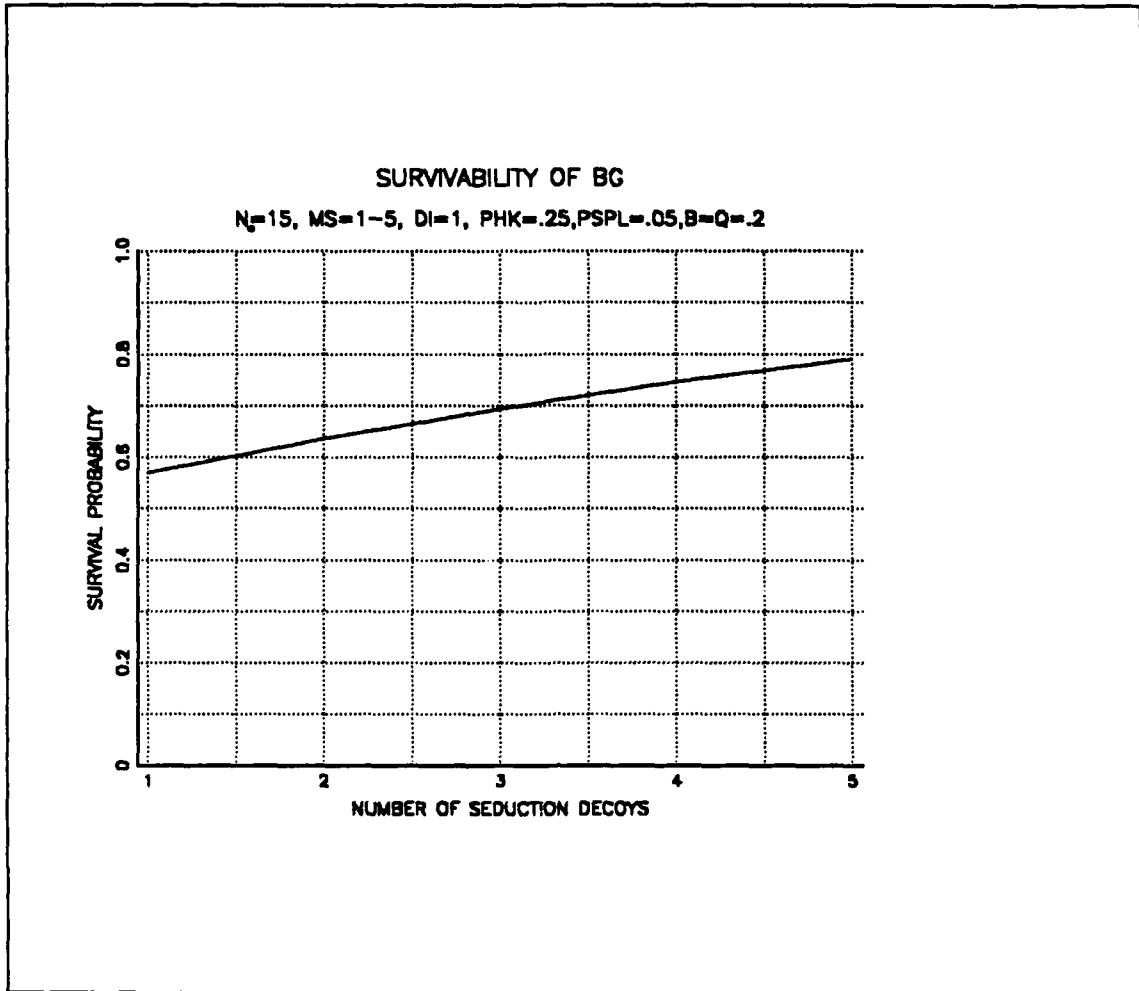
Figure 13.   Survivability of BG while num.of seduction decoys varies

This assumes independence for each missile and that the decoys are not depleted during successive attacks. After running the program and calculating the survival probabilities for M missiles we got the graphs in Figure 14 and Figure 15. In Figure 14 we run the program for M missile attacks while,

1. Number of missile attacking the BG (M) varies from 1 to 10.
2. Number of distraction decoys (DI) varies 1 to 5.
3. Number of seduction decoys (MS) = 1.
4. The number of ships (N_0) = 10.
5. Probability of hard kill level of BG (PHK) = 0.70.
6. Splash probability of the missile (PSPL) = 0.05.

SURVIVABILITY OF BG IN M=1~N MISSILE ATTACKS

N=10, DI=1-5, MS=1, B=.5, Q=.7, PHK=.7, PSPL=.05
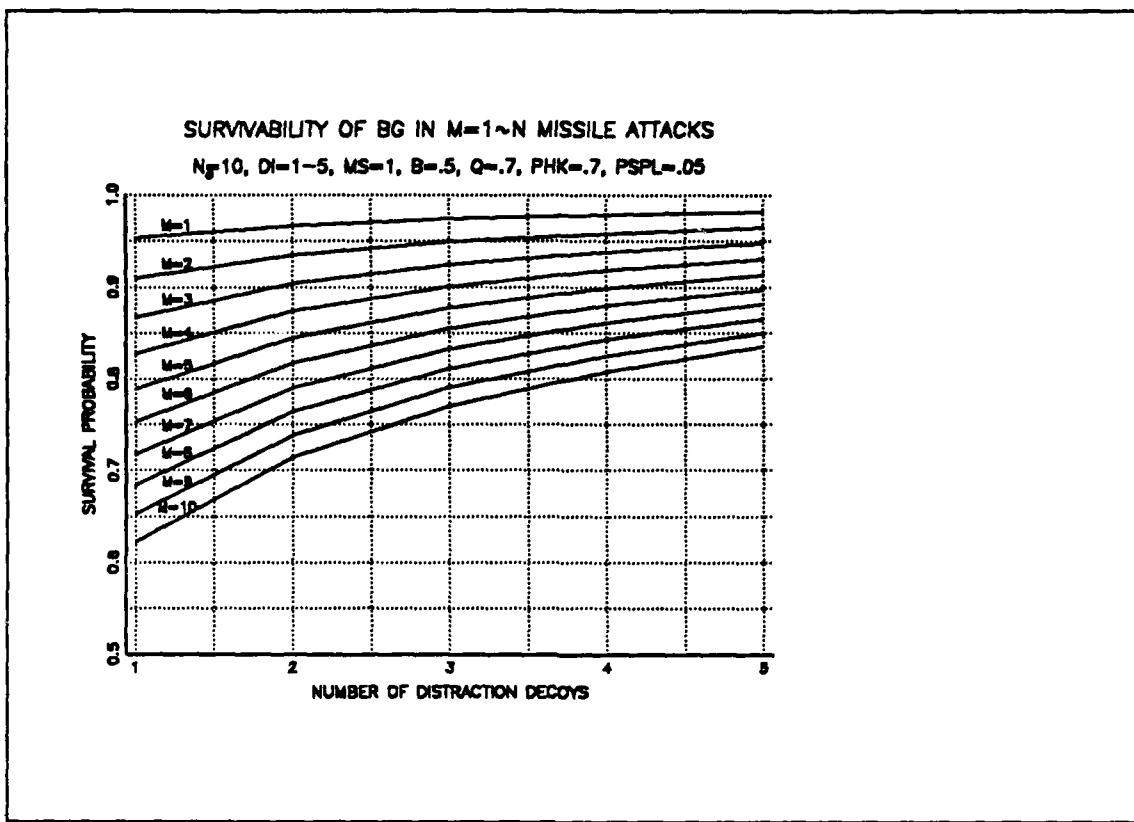
Figure 14. Survivability of BG while num.of distraction decoys varies

7. B factor of seduction decoys = 0.5.

8. Q factor of distraction decoys = 0.7.

9. Alpha angle of the missile $(\alpha) = 15°$.

In Figure 15 we run the program for M missile attacks making the following changes:

1. Number of seduction decoys (DI) varies 1 to 5.

2. Number of distraction decoys (MS) = 1.

From Figure 15 and 16, we can see that seduction decoys have more effect on survivability than distraction decoys have. Especially after the third decoy has been deployed. For each type of decoy there is not a very big difference on survivability for number of attacking missilies (M) = 1, but when M increases the seduction decoys have more effect yielding significantly higher survivability.

The specific values of Q and B are important factors for determining the effect of expendibles. Figures 16 and 17 show the effects of varying the Q factor of a dis-

SURVIVABILITY OF BG IN M=1~N MISSILE ATTACKS

N_0=10, MS=1-5, DI=1, B=.5, Q=.7, PHK=.7, PSPL=.05
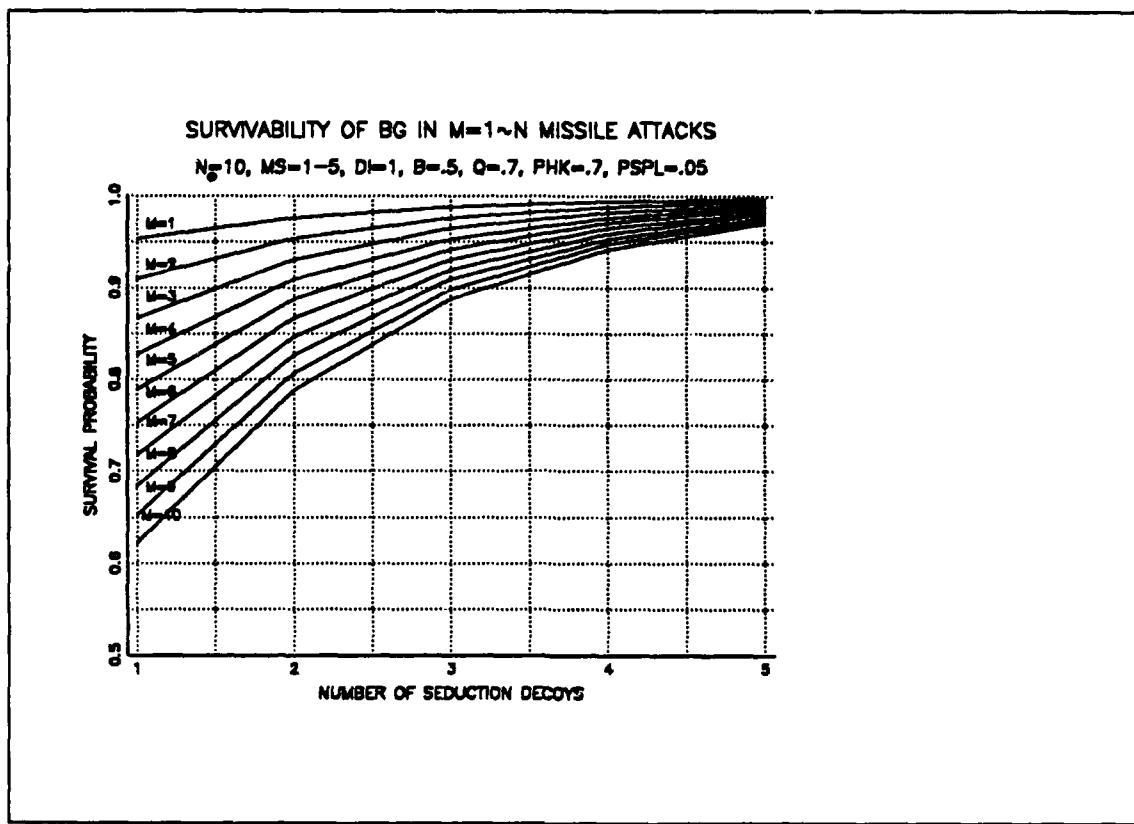
Figure 15.   Survivability of BG while num.of seduction decoys varies

traction decoy and the B factor of a seduction decoy, for the case: 1 seduction and 1 distraction expended for engagement by 1 missile. In each case B and Q varied between 0.0 to 1.0 while the other parameters remained unchanged. The effect of seduction decoy B factor to survivability is higher than the effect of distraction decoy Q factor. In surface ships Q and B may assume any value between 0 to 1. The specific values of B and Q depend on systems employed and the dynamics of the engagement. In Figure 16, we run the program for different values of distraction decoy Q factor while other parameters remain unchanged.

1. Q factor of distraction decoys varies from 0 to 1.

2. Number of seduction decoys (MS) = 1.

3. The number of ships (N_0) = 15.

4. Number of distraction decoys (DI) = 1.

5. Probability of hard kill level of BG (PHK) = 0.25.

6. Splash probability of the missile (PSPL) = 0.05.

SURVIVABILITY OF BG

N_s=10, Q=0~1, B=.2, DI=MS=1, PHK=.25, PSPL=.05
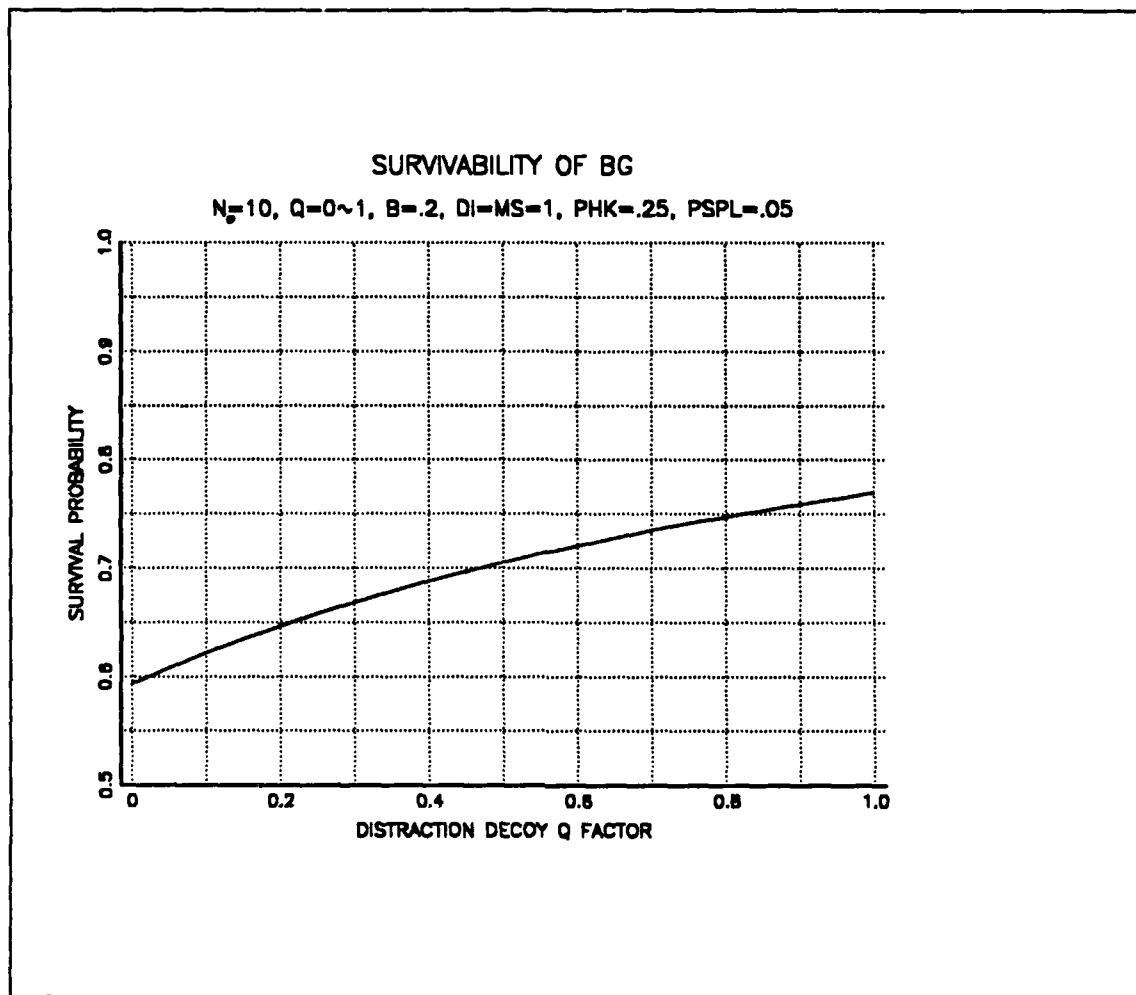
Figure 16.   Survivability of BG while Q factor of distraction decoys varies

7. B factor of seduction decoys = 0.2.

8. Alpha angle of the missile (α) = 15°.

In Figure 17 we run the program for different values of the seduction decoy B factor while other parameters stay unchanged.

1. B factor of seduction decoys varies from 0 to 1 (0.1 increments).

2. Q factor of distraction decoys = 0.2.

30
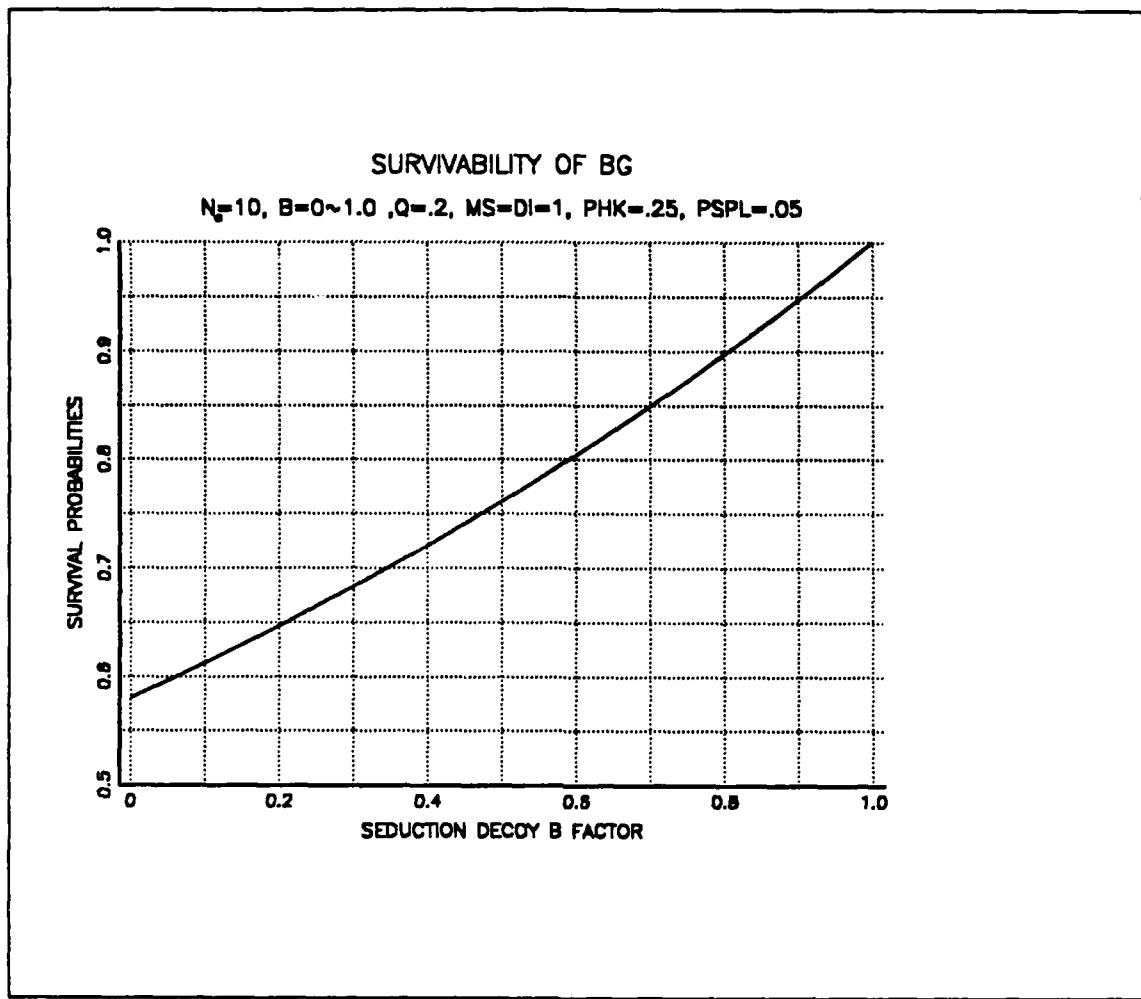
Figure 17. Survivability of BG while B factor of seduction decoys varies

IV. CONCLUSION

While defending the BG against anti-ship cruise missile attacks in the open ocean, a major concern is survivability of the BG and the impact of the different performance factors on the survivability of the BG.

Since there is a big difference between the speed factors of a BG and a missile, we have assumed that the BG's location is fixed in this particular situation. We also defined the measure of effectiveness (MOE) as the probability that all ships survive the attack of M missiles.

Major factors which impact the BG's survivability are number of ships in the BG, the BG's hard kill level, and the BG's expendable decoys. We have seen the benefit of expendables in the examples of the previous chapter which were drawn using the data calculated by the FORTRAN 77 computer program in the Appendix. Specifically, we have investigated the sensitivity to the distraction decoys Q factor and the seduction decoys B factor which are important for determining the effects of expendables.

The analytic results of the model which were calculated by the computer program give results consistent with those expected in the beginning. If the number of ships increases and the other factors remain constant, survivability of BG / ship decreases, in part because of the existence of the nonlinear interactions between ships in the BG. The tactics of single ship defense against ASCM's, the position of the ship in the BG (the ship's nearness to center / high-value ships) may also affect the BG's multiple ship overall survivability, or the survivability of high value targets such as CV's.

Different types of decoys affect survivability differently. This is done by either the B or Q values of the different type of decoys, and / or the number of decoys that can be deployed. Specifically distraction decoys' Q factor and seduction decoys' B factor are

32

important for determining the effects of expendables. Hence one should analyze survivability of the BG and include cost factors of the expendables in the model before allocating and deploying them on each ship. Different trade-offs also can be made between decoys, hard kill levels, and number / position of ships in the BG, which has limited space and budget for devices, weapons, amount of decoys, and missiles.

When we include the cost factors of these variables in our model we can easily get trade-offs by using a survivability cost comparision between different types of assets. By using the results of this model we can change the different variables such as the number of decoys and hard kill levels, subject to given cost constraints, to optimize the survivability.

APPENDIX   A. FORTRAN PROGRAM TO CALCULATE THE SHIP

SURVIVABILITY IN BG

```
***************************************************************
*                                              21,AUG,1988 *
*        PROGRAM SENGEL                            MOD7     *
*        AUTHOR. CENGIZ SENGEL                             *
*        NPGS OPERATIONS RESEARCH.( 360 )                  *
*        ***********                                       *
*   THIS FORTRAN 77 PROGRAM CALCULATES THE SHIP SURVIVAL PROBABILITY  *
* IN THE BATTLE GROUP ( BG ) FOR N GIVEN NUMBER OF SHIPS AND OTHER    *
* PARAMETERS WRITTEN BELOW. PROGRAM ALSO CALCULATES THE SHIP          *
* HIT FOR BG, SINGLE SHIP HIT / SURVIVAL, ATTACKING MISSILE,          *
* DIVERSION, KILL PROBABILITIES FOR M MISSILE ATTACKS. ASSUMING       *
* INDEPENDENT MISSILE ATTACKS M CAN BE 1,2,...,N AND BG HAS ALMOST     *
* CIRCULAR FORMATION AGAINST MISSILE THREAT WHICH MAY COME FROM        *
* ANY DIRECTION.                                                      *
*        ***********                                       *
*   PARAMETERS:                                            *
*        N= # OF SHIPS/INPUT                               *
*        L=MAXIMUM LAYERS FOR GIVEN N (REAL)               *
*        MXL= L VALUE ROUND TO NEAREST INTEGER             *
*        TSUM= TOTAL PROBABILITIES BETWEEN .5 AND L        *
*        TS  = TOTAL PROBABILITIES WITH USING 1-PROBABILITY BETWEEN 0 *
*               AND .5                                     *
*        TSUM= TOTAL PROBABILITY AREA UNDER PDF FCN BETWEEN 0.5 ,MXL  *
*        A=ALPHA ANGLE OF ONCOMING MISSILE/INPUT           *
*        Q= QUALITY FACTOR OF DISTRACTION DECOY (Q.LE.1)/INPUT        *
*        D= # OF DISTRACTION DECOYS/INPUT                  *
*        B= PROBABILITY OF BREAKING THE LOCK FOR A SEDUC. DECOY/INPUT *
*        MS= # OF SEDUCTION DECOYS/INPUT                   *
*        PHK= HARD KILL PROBABILITY OF SHIP FOR KILLING  MISSILE/INPUT*
*        PSPL=PROBABILITY OF MISSILE GET SPLASHED AFTER DECOYED/INPUT *
*        PSH= PROBABILITY OF SHIP GET HIT                  *
*        PDV= PROBABILITY OF MISSILE GET DIVERTED          *
*        PMK= PROBABILITY OF MISSILE GET KILLED ( HARD KILL OR SPLASH)*
*        CPSS= CONDITIONAL PROBABILITY OF SURVIVAL FOR BG ( PROB. OF  *
* NO HIT)                                                  *
*        CPDV= CONDITIONAL PROBABILITY OF MISSILE DIVERSION           *
*        CPMK= CONDITIONAL PROBABILITY OF MISSILE KILL                *
*        CPSS1= CONDITIONAL PROBABILITY OF SINGLE SHIP SURVIVABILITY  *
* IN THE BG                                                *
*        CPSH= CONDITIONAL PROBABILITY OF SHIP HIT IN THE BG          *
*        CPSH1= CONDITIONAL PROBABILITY OF SINGLE SHIP HIT PROBABILITY*
* IN THE BG                                                *
*        CPSSM= CONDITIONAL PROBABILITY OF SHIP SURVIVABILITY IN CASE *
* OF M MISSILE ATTACKS                                     *
*        ********************                              *
***************************************************************
        REAL N,L,MXL,P,PRO1,PL,MXL1,PO,PI,TSUM,TS,A,Q,D,B,MS,PSPL,PHK,PSHI
     5 ,PSH,PDV,M,PDVI,PMKI,PMK,DI,MXL2,PL1,PLO,AL,CPSS,CPSH,CPSS1,CPSH1,
```

```
      6CPSS1M,CPSH1M,K,R
       CALL EXCMS ('FILEDEF 01 DISK DEMET OUTPUT A1')
       DATA N/10/,A/15./,PSPL/.05/,PHK/.7/,DI/1./,Q/.7/,B/.5/
* DEFINING THE CONSTANT PI AND ALPHA RADIAN
       PI=3.141592654
       AL=-2*(A*(PI/180.))
       DO 14 R=1.,5.
*      DO 14 N=5., 50.
       M=N
       MS=R
*      WRITE(01,24) N
24     FORMAT(1X,F5.1)
*CALCULATING THE MAXIMUM NUMBER OF LAYERS
       L=2*SQRT(N/PI)
*ROUNDING THE MAXIMUM NUMBER OF LAYERS TO THE NEAREST INTEGER
       MXL=ANINT(L)
       SUM=0
       MXL1=(MXL-1)+.5
*CALCULATION OF THE AREA UNDER PDF=TS (.5,L) AND AREA OF LAST PORTION
       P0=ABS(1-SQRT(1-(.5**2*PI/(4*N))))
       PL=(SQRT(1.-(L**2.*PI/(4.*N))))-(SQRT(1.-(MXL1**2.*PI/(4.*N))))
       TS=1-P0
       PLP=ABS(PL)/TS
*      WRITE(01,22) PLP,L,TS
22     FORMAT( 1X,F9.8,1X,F9.6,1X,F9.8)
       T=0.0
       CPSH=0.0
       CPMK=0.0
       CPDV=0.0
       PSH=0.0
       PMKI=0.0
       PDV=0.0
*CALCULATING THE PROBABILITIES FOR ONE LAYER YOU CAN FIND PMK BY TWO WAY
       PSHI=(1-EXP(AL))*((1/(1+(Q*DI)))*((1-B)**MS)*(1-PHK))
       PDIV=(1-EXP(AL))*(((Q*DI)/(1+(Q*DI)))*(1-PHK)*(1-PSPL)
      1+((1-(1-B)**MS)*(1-PHK)*(1-PSPL)*(1/(1+(Q*DI)))))+
      2((EXP(AL))*(1-PHK))
*FIRST WAY OF FINDING PMK
*      PMK=1-(PDIV+PSHI)
*OR SECOND (LONGER WAY) OF FINDING PMK
       PMK1=((1-EXP(AL))*((Q*DI)/(1+(Q*DI)))*(PHK))
       PMK2=((1-EXP(AL))*((Q*DI)/(1+(Q*DI)))*(1-PHK)*PSPL)
       PMK3=(1-EXP(AL))*(1/(1+(Q*DI)))*(1-(1-B)**MS)*PHK
       PMK4=((1-EXP(AL))*(1/(1+(Q*DI)))*(1-(1-B)**MS)*(1-PHK)*PSPL)
       PMK5=((1-EXP(AL))*(1/(1+(Q*DI)))*((1-B)**MS)*PHK)+EXP(AL)*PHK
       PMK6=EXP(AL)*PHK
       PMK=PMK1+PMK2+PMK3+PMK4+PMK5
*CALCULATION OF PROBABILITIES FOR LAYERS
          DO 15 F=1,MXL,1
            PR=0.0
            PSH=(PSH+PDIV**(T)*PSHI)
            T=T+1
            PDV=PDIV**F
            IF(F.EQ.MXL)THEN
            CPSH=CPSH+PSH*PLP
            CPMK=CPMK+PMKI*PLP
```

```fortran
              CPDV=CPDV+PDV*PLP
              ELSE
              PR=SQRT(1-((F+.5)**2*PI/(4*N)))-SQRT(1-((F-.5)**2*PI/(4*N)))
              PRO1=ABS(PR)/TS
              PMKI=(1-(PDV+PSH))
              CPSH=CPSH+PSH*PRO1
              CPMK=CPMK+PMKI*PRO1
              CPDV=CPDV+PDV*PRO1
              END IF
*             WRITE(01,27)DI,MS
27            FORMAT( 1X,2(F9.6,1X))
15      CONTINUE
        CPSS=1-CPSH
*DISPLAY CONDITIONAL PROBABILITY OF SHIP HIT, MISSILE KILL, DIVERSION
*       WRITE(01,23)CPSH,CPMK,CPDV
*DISPLAY THE SHIP SURVIVABILITY AS 'MOE'
*SINGLE SHIP PROBABILITY CALCULATIONS
*       CPMK1=CPMK/N
*       CPSH1=CPSH/N
*       CPSS1=CPSS/N
*BG PROBABILITY CALCULATIONS GIVEN M MISSILE ATTACKS TO THE BG
        CPSHM=1.-(1.-(CPSH)**M)
        CPSSM=1.-(1.-(CPSS)**M)
*SINGLE SHIP PROBABILITY CALCULATIONS GIVEN M MISSILE ATTACKS TO THE BG
*WHICH IS M = 1, 2, 3, ..., N.
        CPSH1M=1.-(1.-(CPSH/M)**M)
        CPSS1M=1.-(1.-(CPSS/M)**M)
        WRITE(01,23)N,MS,DI,CPSS,CPSSM
23      FORMAT( 1X,F4.1,1X,F4.1,1X,F4.1,1X,2(F9.7,1X))
14      CONTINUE
31      STOP
        END
```

APPENDIX B. PROBABILITY CALCULATIONS FOR ONE LINE OF ENCOUNTERS.

We can obtain all the equations calculated for one line of encounters with the aid of the probabilities on the probability tree diagram in Figure 18.

For calculation of the total missile kill probability (TPMK):

$$PMK_1 = (1 - e^{-2\alpha})(\frac{QD}{(1 + QD)})PHK \ ,$$

$$PMK_2 = (1 - e^{-2\alpha})(\frac{QD}{(1 + QD)})(1 - PHK)PSPL \ ,$$

$$PMK_3 = (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})(1 - (1 - B)^{MS})PHK \ ,$$

$$PMK_4 = (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})(1 - (1 - B)^{MS})(1 - PHK)PSPL \ ,$$

$$PMK_5 = (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})(1 - B)^{MS}PHK \ ,$$

$$PMK_6 = e^{-2\alpha}PHK \ ,$$

$$TPMK = PMK_1 + PMK_2 + PMK_3 + PMK_4 + PMK_5 + PMK_6$$

$$TPMK = (1 - e^{-2\alpha})(\frac{QD}{(1 + QD)})PHK + (1 - e^{-2\alpha})(\frac{QD}{(1 + QD)})(1 - PHK)PSPL$$

$$+ (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})(1 - (1 - B)^{MS})PHK + (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})$$
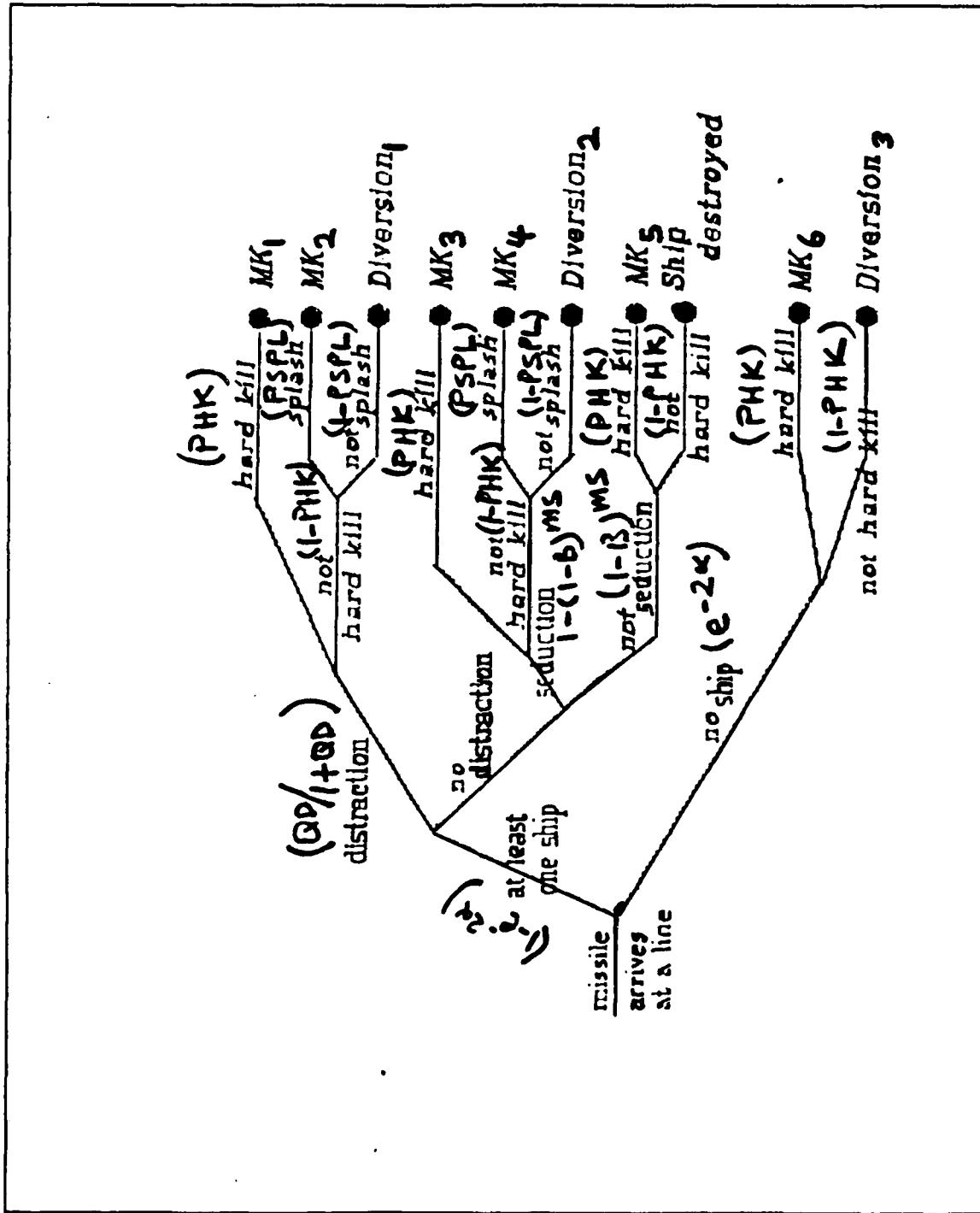
Figure 18.   Probability calculations for one line of encounters.

$$(1 - (1 - B)^{MS})(1 - PHK)PSPL + (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})(1 - B)^{MS}PHK$$

$$+ (e^{-2\alpha})PHK \ .$$

For calculation of the total missile diversion probability (TPDIV):

$$PDIV_1 = (1 - e^{-2\alpha})(\frac{QD}{(1 + QD)})(1 - PHK)(1 - PSPL) \ ,$$

$$PDIV_2 = (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})(1 - (1 - B)^{MS})(1 - PSPL) \ ,$$

$$PDIV_3 = e^{-2\alpha}(1 - PHK) \ ,$$

$$TPDIV = PDIV_1 + PDIV_2 + PDIV_3 \ .$$

$$TPDIV = (1 - e^{-2\alpha})(\frac{QD}{(1 + QD)})(1 - PHK)(1 - PSPL) + (1 - e^{-2\alpha})(\frac{1}{(1 + QD)}) \ .$$

$$(1 - (1 - B)^{MS})(1 - PSPL) + (e^{-2\alpha})(1 - PHK) \ .$$

For calculation of the ship hit probability and ship survival probability:

$$PSHI = (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})(1 - B)^{MS}(1 - PHK) \ ,$$

$$PSSUR = (1 - (1 - e^{-2\alpha})(\frac{1}{(1 + QD)})(1 - B)^{MS}(1 - PHK)) \ .$$

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